USN | | | | | | | | | | | **06EC65**

Sixth Semester B.E. Degree Examination, Dec.2014/Jan.2015

Information Theory and Coding

Time: 3 hrs. Max. Marks: 100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. A binary source is emitting an independent sequence of '0's and is with probabilities of p and (1 – p) respectively. Plot the entropy of the source versus p. **(06 Marks)**

b. Design a system to report the heading of a collection of 800 cars. The heading levels are: Heading straight (S), turning left (L) and turning right (R). This information is to be transmitted every second. Construct a model based on the test data given below:

1. One the average during a given reporting interval, 400 cars were heading straight, 200 were turning left and remaining were turning right.
2. Out of 400 cars that reported heading straight, 200 of them reported going straight during the next reporting period, 100 of them turning left and remaining turning right during the nest period.
3. Out of 200 cars that reported as turning during a signalling period, 100 of them continued their turn and remaining headed straight during the next reporting period.
4. The dynamics of car did not allow them to change their heading from left to right or right to left during subsequent reporting periods.

i) Find the entropy of each state.
ii) Find the entropy of the system.
iii) Find the rate of transmission. **(10 Marks)**

c. Consider a discrete memoryless source with source alphabet s = $\{s_0, s_1, s_2\}$ with source statistics {0.7, 0.15, 0.15}.

i) Calculate the entropy of the source.
ii) Calculate the entropy of the second order extension of the source. **(04 Marks)**

2 a. The state diagram of markoff source is shown in Fig.Q2(a):

i) Find the entropy H of the source.
ii) Find G_1, G_2 and verify that $G_1 > G_2 > H$.

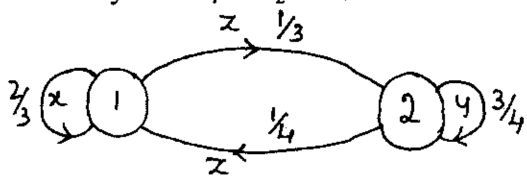

Fig.Q2(a) **(10 Marks)**

b. Explain the properties of codes with example. **(05 Marks)**

c. Construct binary code for the following source using Shannon's binary encoding procedure:
S = $\{s_1, s_2, s_3, s_4, s_5\}$ P = {0.4, 0.25, 0.15, 0.12, 0.08} **(05 Marks)**

3 a. Consider a source with 8 alphabets A to 4 with respective probabilities of 0.22, 0.20, 0.18, 0.15, 0.10, 0.08, 0.05, 0.02.

i) Construct a binary compact (Huffman) code and determine the code efficiency.
ii) Construct a quarternary compact code and determine the code efficiency. **(10 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

b. Define mutual information and list its properties. **(04 Marks)**

c. For the given channel matrix calculate H(x, y), H(y/x) and I(x, y) where $p(x_1)$, $p(x_2)$, $p(x_3)$, p(x) is 0.3, 0.2, 0.3 and 0.2 respectively.

$$P(y/x)=\begin{bmatrix} 1/6 & 0 & 2/3 & 1/6 \\ 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 2/3 & 1/3 \\ 1/4 & 1/4 & 0 & 1/2 \end{bmatrix}$$ **(06 Marks)**

4 a. Two noisy channels are cascaded whose channel matrixes are given by

$$P(y/x)=\begin{bmatrix} 1/5 & 1/5 & 3/5 \\ 1/2 & 1/3 & 1/6 \end{bmatrix} \text{ and } P(z/y)=\begin{bmatrix} 0 & 3/5 & 2/5 \\ 1/3 & 2/3 & 0 \\ 1/2 & 0 & 1/2 \end{bmatrix}$$

with $p(x_1) = p(x_2) = 1/2$. Find the overall mutual information I(x, z) **(10 Marks)**

b. State and explain Shannon-Hartley law and derive an expression for the capacity of the noisy channel. **(06 Marks)**

c. A Gaussian channel has a 10 MHz B.W. If S/N ratio is 100, calculate the channel capacity and the maximum information rate. **(04 Marks)**

PART – B

5 a. For a systematic (7, 4) linear block code, the parity matrix p is given by:

$$[p]=\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

i) Find all possible valid code vector.

ii) Draw the corresponding encoding circuit.

iii) A single error has occurred in each of these received vectors. Detect and correct those errors.

1. R_A = [0111110] 2. R_B = [1011100] **(10 Marks)**

b. The parity check bits of a (7, 4) hamming code are generated by $c_5 = d_1 + d_3 + d_y$, $c_6 = d_1 + d_2 + d_3$, $c_7 = d_2 + d_3 + d_4$ where d_1, d_2, d_3 and d_4 are message bits.

i) Find the generator matrix [G] and parity check matrix [H] for this code.

ii) Prove that $GH^T = 0$.

iii) The (n, k) linear block code so obtained has a dual code. The dual code is a (n, n – k) code having a generator matrix G and parity check matrix H. Determine the eight code vectors of the dual code for (7, 4) Hamming code described above.

iv) Find the minimum distance of the dual code determined in part c. **(10 Marks)**

6 a. Obtain the code vector for (7, 3) expurgated hamming code and construct a encoder for it given $g(x) = (1 + x^2 + x^3)$. **(08 Marks)**

b. The generator polynomial for a (15, 7) cyclic code is $g(x) = 1 + x^4 + x^6 + x^7 + x^8$.

i) Find the code-vector in systematic form for the message $D(x) = x^2 + x^3 + x^4$.

ii) Assume that 1st and last bit of the code vector V(x) for $D(x) = x^2 + x^3 + x^4$ suffer transmission errors. Find the syndrome of V(x). **(08 Marks)**

c. What are cyclic codes? Mention its properties. **(04 Marks)**

7 a. For the convolutional encoder shown in Fig.Q7(a):

i) Find the impulse response and hence calculate the output produced by the information sequence 10111.

ii) Write the generator polynomial of the encoder and recomputed the output for the input of (i) and compare with that of (i).

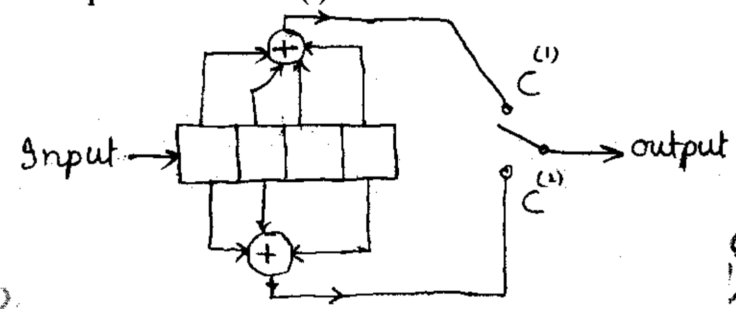

Fig.Q7(a) **(08 Marks)**

b. Consider the convolutional encoder shown in Fig.Q7(b). The code is systematic:

i) Draw the state diagram and state transition table.

ii) Draw the code tree.

iii) Find the encoder output produced by the message sequence 10111.

iv) Verify the output using time-domain approach.

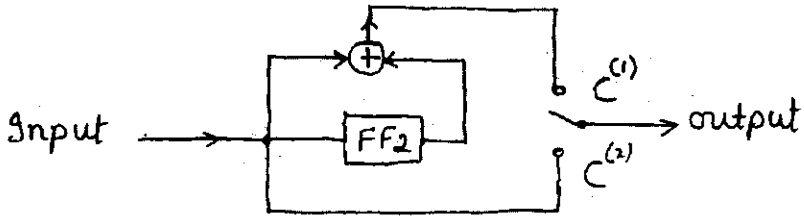

Fig.Q7(b) (2, 1, 1) convolutional encoder circuit **(12 Marks)**

8 Write short notes on:

a. Golay codes

b. Reed Solomon codes

c. Burst and random error correcting codes

d. BCH codes **(20 Marks)**

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